Substitution amoung Labor, Capital and Energy for United States Manufacturers

SENIOR THESIS

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by

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Abstract

A three input translog cost function in labor, capital and energy was estimated for 298 U.S. manufacturing industries for the years 1972 and 1976. A statistically significant change in the function was observed between these years indicating a change in technology occurred. The Allen Partial elasticity of substitutions for the inputs were calculated and the substitution possibilities between labor and energy and capital and energy had become easier. These calculations were repeated for subgroups of industries divided by fuel cost share, and for industries with 1% to 3% fuel cost share similar results occurred. For other subgroups, no change in technology was observed, and no pattern was apparent in the elasticities.

I. Introduction

The manner in which manufacturing industries faced the recent increases in the price of energy is important to governmental decision makers and the public in general. From the general press(Shabecoff, 1980), to economic tracts (Slesser, 1978), individuals are trying to form coherent views on how energy price changes will filter through the economy. This paper attempts to accertain the effects on the substitution between the factors of production, fuel, capital, and labor in United States manufacturing firms.

Humphrey(1975) studied the substitution possibilities between labor, capital and natural resources, leading to a better understanding of how industries cope with scarce resources. Slesser(1978) discusses the engineering problems associated with energy change, and analyzes flaws in previous economic arguments that had omitted energy considerations.

Berndt(1975) studied the substitution possibilities between labor, capital, fuel and other secondary inputs, for U.S. manufacturers. He used Divisia indices to obtain observations for each year. The results from this study show the ease of substitution between these inputs has remained nearly constant from 1948 to 1971.

Manufacturing industries provide an especially fruitful ground for the study of the effects of energy price changes. In contrast to the Berndt study, this study estimates a cost function for industry groups in the years 1972 and 1976. These two cost functions will be compared yielding information on technology change. Elasticities of substitution between the inputs will be calculated giving information on the relative ease of substitution between the inputs. It is not clear that a technology change will have occurred in the short span of four years. If it has, it has been suggested that firms will move to more elastic positions with respect to ease of sub substitution between fuel and other inputs. The reason is that firms have experienced an unexpected fuel price shock. If they expect more such shocks, they will want to have as much flexibility in their future input combination choices as possible, hence the elasticity should increase. If the technology has not changed, then a change in elasticity may occur due to change in price, but the direction of the change will depend on the industry.

Section II contains a brief review of the production theory underlying the cost function approach used here. The data, along with the estimation procedure and statistical results are described in the third section. Section IV concludes.

II. Theory

A production function describes the relationship between the inputs a firm uses and the output it produces. Neoclassical production theory predicts that output is a function of inputs. The theory also provides measures of the relationships between the inputs and the technology used.

The Allen Partial Elasticity of Substitution,(Allen, 1938) provides a standard measure of how easy it is for a firm to move from one set of input configurations to another under constant output. This gives the ease with which production output can be maintained by substituting one input for another. Allen has shown that if the elasticity (σ_{ij}) between inputs i and j is greater than zero, then the usage of factor i increases as that of factor j decreases, so the two factors are substitutes in the sense that either of them can be used in place of the other. If $\sigma_{ij} < 0$ then a decrease in the use of factor j requires a decrease in factor i, to hold output constant and so the factors are complements. Allen has also shown that the competive mode, with the inputs as substitutes dominates.

Another approach to production is from the cost side. For any particular set of input prices and output level, there will be an associated cost function

$$C = c(Y, P_1, P_2, \dots P_{n-1})$$

where

Y is output
C is cost
P_i is the price of i^{th} input.

Under the following assumptions on C, first given by Shepard(1953), there is a well defined production function which is the dual to this cost function. The conditions require that: (1). C is minimal for a given set of inputs. (2). C is continuously differentiable with respect to P_i.

(3). C is a positive real valued continuous function tending to infinity as Y tends to infinity. (4). C is linearly homogeneous in P_i. (5). C is concave in P_i. If these restrictions are satisfied, the cost function and the production function provide the same information about the technology.

Uzawa(1962) showed that the elasticity of substitution between inputs i and j in terms of the cost function is

$$\sigma_{ij} = \frac{c \frac{\partial^2 c}{\partial P_i \, \partial P_j}}{\frac{\partial c}{\partial P_i} \; \frac{\partial c}{\partial P_j}} + 1.$$

Production theory, or its cost equivalent, provides limited guidance about the functional form of cost functions. Unitl recently, the most common forms have been the Cobb-Douglas (Douglas, 1928) and the Constant Elasticity of Substitution (CES) form (Arrow, Chenery, Minhas, Solow, 1961). Both of these functional forms can place severe restrictions on a cost model.

The technical difficulties with restrictive forms such as these are detailed in Christensen(1973). The problem most relevant to this work is that relating to the elasticity of substitution. The Cobb-Douglas form implies the elasticity of substitution is unity, which makes studies like this impossible. The CES form requires the elasticity of substitution to be constant throughout the production space. This would require the elasticity of substitution between say labor and capital to be the same for the steel making industry and the construction industry, which seems overly restrictive.

Other forms have been developed, but still require an a priori choice of functional form. Christensen(1971,1973) developed a second order Taylor expansion approximation to both the cost and production functions. This form, known as the translog function, places no stronger restrictions on the production function than imposed by general production theory. In particular, the elasticity of substitution can vary at each point. Of course, a decision has to be made as to the necessary order of approximation, but this is a fundamentally different problem from that of deciding what functional form should be used for a model. The translog cost function is used here because: (1). Price data do not require aggregation indexes. (2). Statistical analysis of the elasticities of substitution is presently possible only for the cost side. (3). Studies on the efficency of the regression have been done for the cost side.

The translog function is the second order Taylor polynomial approximation of an arbitrary function in the logarithm of a normalized form of the variables. Normalized means that the variables are divided by some mean, eg. geometric, or arithmetic.

If

$$\ln C = f(\overline{Y}, \overline{P}_1, \ldots \overline{P}_{n-1})$$

is the log of the cost function, where the barred variables are the actual values, then a normalized functional form is

$$\ln C = h(Y, P_1, P_2, \ldots P_{n-1})$$

where

$$Y = \frac{\overline{Y}}{Y^*} \quad \text{and } Y^* \text{ is the mean of } \overline{Y}, \text{ and}$$

$$P_i = \frac{\overline{P}_i}{P_i^*} \quad \text{and } P_i^* \text{ is the mean of } \overline{P}_i.$$

The final form is in terms of the logarithms of the normalized variables

$$\ln C = c(\ln Y, \ln P_1, \ln P_2, \ldots \ln P_{n-1}).$$

Now that the final form of the function is established, it is necessary to derive its Taylor expansion, The n-dimensional

Taylor expansion around the vector a is

$$c(a + h) = c(a) + \nabla c(a)\cdot h + \tfrac{1}{2}h'\cdot \nabla^2 c(a)\cdot h$$

where

$$\nabla c = (c_1, c_2, \ldots c_n)$$

with

$$c_i = \frac{\partial c}{\partial \ln Y} \quad \text{for } i = 1, \text{ and } \quad c_i = \frac{\partial c}{\partial \ln P_{i-1}} \quad \text{for } i \neq 1$$

and

$$\nabla^2 c = \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{n1} \\ c_{21} & c_{22} & \cdots & c_{n2} \\ \vdots & \vdots & & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}.$$

where the double subscripts denote second derivatives.

The necessity of the functional form changes is now evident. Consider the expansion of c around the vector 0. This is exactly the expansion around the mean of the data, since c(0) occurs when

$$\ln P_i = 0 \quad \text{or} \quad P_i = 1$$

but $P_i = 1$ only when $\overline{P}_i = P_i^*$. So expanding c around 0 is the same as expanding the original function around the mean.

In the theory of polynomial approximation, it is shown that the best fit of the approximation occurs in the region near the point of expansion. Expanding around the mean of the data points and then trying to fit points that are the log of the normalized variable distance from the mean, results in a closer fit than expanding around other less related points. Econometric considerations also agree with this technique. Thursby and Lovell(1978) have shown that increasing the number of observations in the region of convergence of an approximation improves the fit of a regression.

The Taylor expansion around 0 with

$$h = (\ln Y, \ln P_1, \ln P_2, \ldots \ln P_{n-1})$$

is of the form

$$c(h) = c(0) + \nabla c(0)\cdot h + \tfrac{1}{2}h'\cdot \nabla^2 c(0)\cdot h,$$

which leads immediately to the non-matrix form for the translog cost function

$$\ln C = c_o + c_y \ln Y + \tfrac{1}{2}c_{yy}(\ln Y)^2 + \sum_i c_i \ln P_i + \tfrac{1}{2}\sum_i \sum_j c_{ij} \ln P_i \ln P_j + \sum_i c_{yi} \ln Y \ln P_i$$

where

$$c_o = c(0) \qquad \text{and } c_{ab} = c_{ab}(0).$$

It should be noted that $c_{ij} = c_{ji}$ for the well-defined cost function.

A well-behaved cost function satisfies several restrictions. The first is first degree honogeneity in prices. Using proportionate factor change b, it must be true that

$$\ln C(bP_i, Y) = \ln b + \ln C(P_i, Y).$$

For the translog case,

$$\ln C(bP_i, Y) = c_o + c_y \ln Y + \tfrac{1}{2}c_{yy}(\ln Y)^2 + \sum_i c_i \ln bP_i + \sum_i \sum_j c_{ij} \ln bP_i \ln bP_j + \sum_i c_{yi} \ln Y \ln bP_i$$

$$\begin{aligned}
&= c_o + c_y \ln Y + \tfrac{1}{2} c_{yy} (\ln Y)^2 + \\
&\quad \sum_i c_i \ln b + \sum_i c_i \ln P_i + \\
&\quad \tfrac{1}{2} \sum_i \sum_j c_{ij} \ln b \ln P_i + \tfrac{1}{2} \sum_i \sum_j c_{ij} \ln b \ln b \\
&\quad \tfrac{1}{2} \sum_i \sum_j c_{ij} \ln b \ln P_i + \tfrac{1}{2} \sum_i \sum_j c_{ij} \ln P_i \ln P_j + \\
&\quad \sum_i c_{yi} \ln Y \ln b + \sum_i c_{yi} \ln Y \ln P_i .
\end{aligned}$$

Hence

$$\begin{aligned}
\ln Y + \ln b = \ln b \sum_i c_i + \tfrac{1}{2} (\ln b)^2 \sum_i \sum_j c_{ij} + \\
\tfrac{1}{2} \ln b \sum_i \sum_j c_{ij} \ln P_j + \tfrac{1}{2} \ln b \sum_i \sum_j c_{ij} \ln P_i + \\
\ln b \sum_i c_{yi} \ln P_i .
\end{aligned}$$

This equation implies the following restrictions

$$\sum_i c_{yi} = 0 \qquad \sum_i c_i = 1 \qquad \sum_i \sum_j c_{ij} = 0$$
$$\sum_i c_{ij} = 0 \qquad \sum_j c_{ij} = 0.$$

These restrictions will be imposed on the regression.

The properties of monotonicity, unboundedness and concavity can be reduced to showing output monotonicity and price monotonicity, i.e. cost increases as output increases and cost increases as input factor prices increase. The first condition states

$$\frac{\partial c}{\partial Y} > 0 \quad \text{or} \quad \frac{\partial \ln c}{\partial \ln Y} > 0 \quad \text{since} \quad \frac{\partial \ln c}{\partial \ln Y} = \frac{Y}{c} \frac{\partial c}{\partial Y}$$
and $Y, c > 0$.

So, for the translog each point must satisfy

$$\frac{\partial \ln c}{\partial \ln Y} = c_y + c_{yy} \ln Y + \sum_i c_{yi} \ln P_i > 0.$$

Similarly, the second condition states

$$\frac{\partial c}{\partial P_i} > 0 \quad \text{or} \quad \frac{\partial \ln c}{\partial \ln P_i} > 0 \quad \text{for the same reason.}$$

Accordingly, for each point

$$S_i = \frac{\partial \ln c}{\partial \ln P_i} = c_i + \sum_j c_{ij} \ln P_i + c_{iy} \ln Y > 0$$

which is also the cost share of input i.

The Allen Partial Elasticity of Substitution can be derived from the expression

$$\frac{\dfrac{\partial^2 \ln c}{\partial \ln P_i \, \partial \ln P_j}}{\dfrac{\partial \ln c}{\partial \ln P_i} \; \dfrac{\partial \ln c}{\partial \ln P_j}}$$

which can be shown to be equal to $\sigma_{ij} - 1$.

Now

$$\frac{\partial \ln c}{\partial \ln P_i} = S_i$$

where S_i is defined above and

$$\frac{\partial S_j}{\partial \ln P_i} = \frac{\partial S_i}{\partial \ln P_j} = \frac{\partial^2 \ln c}{\partial \ln P_i \, \partial \ln P_j} = c_{ij}$$

by simple differentiation. Hence

$$\frac{c_{ij}}{S_i S_j} = \sigma_{ij} - 1 \quad \text{or} \quad \sigma_{ij} = \frac{c_{ij}}{S_i S_j} + 1.$$

These results from production theory are used in the following cost model of U.S. manufacturing industries.

III. Estimation

In this section, a three factor translog cost function is estimated for the factors capital, labor and fuel using the price of capital(K), the price of labor(L), and the price of fuel(F). The stocastic form of the translog is

$$\begin{aligned} \ln c = c_o + c_y \ln Y + .5c_{yy}(\ln Y)^2 + c_L \ln L + \\ c_k \ln K + c_f \ln F + .5c_{LL}(\ln L)^2 + \\ c_{Lk} \ln L \ln K + c_{Lf} \ln L \ln F + .5c_{kk}(\ln K)^2 + \\ c_{kf} \ln K \ln F + .5c_{ff}(\ln F)^2 + c_{Ly} \ln L \ln Y + \\ c_{ky} \ln K \ln Y + c_{fy} \ln F \ln Y + u \end{aligned}$$

where the subscripts referring to observations have been dropped for convenience and u is individually identically distributed $N(0, \sigma^2)$. To avoid ambiguities, the unsubscripted sigmas always represent statistical quantities, such as variances, while the subscripted sigmas represent elasticities.

After normalizing the variables, the output monotonicity condition is

$$c_y + c_{yy} \ln Y + c_{yL} \ln L + c_{yk} \ln K + c_{yf} \ln F > 0$$

and price monotonicity requires

$$S_L = c_L + c_{LL} \ln L + c_{Lk} \ln K + c_{Lf} \ln F + c_{Ly} \ln Y > 0$$

$$S_k = c_k + c_{kL} \ln L + c_{kk} \ln K + c_{kf} \ln F + c_{ky} \ln Y > 0$$

$$S_f = c_f + c_{fL} \ln L + c_{fk} \ln K + c_{ff} \ln F + c_{fy} \ln Y > 0$$

where S_i is the cost share of input i. These conditions held for the mean industries in all the regressions, though between 10% and 15% of the observations on S_k or S_f are negative. This reflects the fact that the translog is in fact an approximation, not that these industries are in noneconomic regions.

The Allen Partial Elasticities of Substitution for this model are

$$\sigma_{Lk} = \frac{c_{Lk}}{S_L S_k} + 1$$

$$\sigma_{Lf} = \frac{c_{Lf}}{S_L S_f} + 1$$

$$\sigma_{fk} = \frac{c_{fk}}{S_f S_k} + 1.$$

For the mean industry, the normalized variables will all equal one, and since ln 1 = 0, S_i reduces to c_i and the elasticities reduce to

$$\sigma_{Lk} = \frac{c_{Lk}}{c_L c_k} + 1$$

$$\sigma_{Lf} = \frac{c_{Lf}}{c_L c_f} + 1$$

$$\sigma_{kf} = \frac{c_{kf}}{c_k c_f} + 1.$$

Single equation least squares is used to estimate the translog cost function. The appropriate restrictions were given earlier. It should be noted that the translog cost function can be estimated in a simultaneous equation setting. This approach has been advocated by Christensen(1973) and Berndt(1975). However, a Monte Carlo study by Guilkey and Lovell(1979) indicates that a single equation restricted least squares regression is slightly more efficent than a simultaneous system estimation.

Writing the translog model as

$$Z = XB + u$$

where Z is a column vector of the logarithms of the costs for the industries, X is the matrix containing the observations on appropriate combinations of variables and u is the disturbance vector. The linear restrictions implied by production theory are

$$RB = r$$

where

$$r = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 000111000000000 \\ 000000111000000 \\ 000000001011000 \\ 000000000000111 \\ 000000010110000 \\ 000000111111000 \end{bmatrix}.$$

Estimation of the model subject to the restrictions is accomplished by Lagrangian Techniques. That is, determine the values of B and λ that minimize

$$W = (Z - XB)'(Z - XB) - \lambda'(RB - r).$$

It is well known that the coefficients estimators become

$$\underline{c} = (X'X)^{-1}Z - 2(X'X)^{-1}R'(R(X'X)^{-1}X'Z + (X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}r.$$

This estimator is normally distributed with mean B and covariance matrix

$$\sigma^2(I - (X'X)^{-1}R'(R(X'X)^{-1}R(X'X)^{-1},$$

under the full ideal conditions on the disturbance term.

The data for this study consists of 298 four digit industries as defined and collected by the United States Census(1971,1972,1976). The years chosen to study the change in technology were 1972 and 1976. The first year was immediately before the first set of energy price increases. 1976 is the latest year data is available on U.S. manufacturers. In Table 1, the changes in the prices of labor, capital and fuel are shown. As can be seen, the price of fuel increased nearly three times as much as the price of labor and increased substantially with respect to capital in this time period.

Statistical estimation of the model requires appropriate

Table 1

Changes in Labor, Capital and Fuel Costs for U.S. Manufacturers 1972, 1976

Year	L($/wkr)	K($\frac{\text{VAD-L}}{\text{Assets}}$)	F($/mBTU)
1972	8895	.6438	.368
1976	12046	.7560	.949
%change	30.0	59.2	88.2

source: U.S. Census(1971,1972,1976)

measures of output, total cost, and prices of fuel, capital and labor. The price of labor was total expenditure on labor divided by the number of employees. To derive a price of capital, assume that labor and capital exhaust value added. This implies constant returns to scale and a homogeneous production function. This is a fairly severe restriction but capital depreciation data of a reliable nature is not available. Humphrey and Moroney(1975) use a similar assumption in a study of the relationship of natural resources, labor and capital. Accordingly, the price of capital is value added minus expenditures on labor divided by total assets.

The obvious choice for price of fuel, expenditure per unit of energy(the units are British Thermal Units) is not immediately available as the Census Bureau reports quantity of energy used for 1976, but reports quantity used in 1971 for the 1972 data series. In order to derive 1972 quantity information, it is necessary to assume that fuel use increased proportionately to value added increases between 1971 and 1972. This can be represented as

$$\frac{VAD_{72}}{VAD_{71}} = \frac{Qfuel_{72}}{Qfuel_{71}}$$

or

$$Qfuel_{72} = \frac{VAD_{72}}{VAD_{71}} Qfuel_{71}$$

where

$Qfuel_{xx}$ is quantity of fuel used in year xx and

VAD_{xx} is value added in year xx.

With this derived value for quantity of fuel used, the price of fuel is expenditure on fuel divided by quantity of fuel used.

Obtaining a reppresentive for output raises a number of difficulties. No generally accepted measure encompassing the

the diverse types of outputs from all manufacturere is available. Value added can be used to represent output as there is a close relationship between quantity of output and value added for a given industry. This measure was adopted.

In obtaining a measure for cost, difficulties arise in determining what quantities should be included as cost. In this study cost is defined as value added plus materials costs plus fuel. Another interesting possibility for further investigation is to examine a two stage production process, where the first stage costs are only value added plus fuel, and the second regresses this variable and materials costs on total costs. The implication of such a procedure is that there is a fixed production relation between materials and all other inputs, hence, this elasticity is zero. The other elasticities are obtained in the first step.

The results of the regressions are shown in Table 2. These statistics implicitly require that the translog form is the exact form of the cost function. This is done so that the distribution of the coefficent estimators and later the derived variances for the elasticities may be obtained. At the present time, it is not known how the disturbance term u and the approximation errors interact, so the distribution of the estimators can not be obtained without this assumption.

From the estimates, it is possible to determine if a change in technology has occurred. If the estimates of the parameters changed significantly, it can be concluded that the underlying cost function has changed, hence the technology has changed. Whether this change has occurred can be determined using a Chow(1961) test where the null hypothesis is that the 1972 and 1976 cost functions are equivalent. The test statistic is

Table 2

Estimates of Translog Parameters, 1972, 1976

Param.	1972 Est.(t-score)	1976 Est.(t-score)
c_o	-.08382(-3.047)	-.1261(-4.725)
c_y	.9239(36)	.9275(38)
c_{yy}	.005829(.217)	-.01644(-.706)
c_L	.8858(19)	.8098(14)
c_k	.08970(2.103)	.02489(.557)
c_f	.02449(.5098)	.1653(2.311)
c_{LL}	.05065(.865)	.2118(2.108)
c_{Lk}	.08395(1.25)	-.1217(-1.724)
c_{Lf}	-.1346(-1.78)	-.09011(-.843)
c_{kk}	-.1346(-1.78)	-.0911(-.843)
c_{kf}	.05065(.866)	.2118(2.108)
c_{ff}	.08395(1.25)	-.1217(-1.72)
c_{Ly}	.06686(1.92)	.05590(1.407)
c_{ky}	-.02264(-.786)	.02509(.6903)
c_{fy}	-.04433(-1.16)	-.08099(-1.41)
	SSE = 37.929 No. data pts. = 298 R^2 = .898	SSE = 33.389 No. data pts. = 298 R^2 = .918

$$F = \frac{(SSE_{(72\&76)} - SSE_{(72)} - SSE_{(76)})/\ \#\ \text{of parameters}}{(SSE_{(72)} + SSE_{(76)})/(\text{total}\ \#\ \text{pts.} - 2(\#\ \text{of parameters}))}$$

where

$SSE_{(xx)}$ is the sum of squared residuals 19xx

$SSE_{(72\&76)}$ is the sum of squared residuals, with all data points(the restricted case).

This statistic is distributed F(# parameters, # pts. -2(# of parameters)). In this case, f = 1.95 and the critical value for $F(15,566)$ at the .05 significance level is 1.67, so the null hypothesis is rejected and it is concluded that a change in technology occurred between 1972 and 1976.

Given that the functions are different for the two years, it is necessary to determine whether the predicted elasticityes changes have occurred. Since the elasticities are nonlinear functions of the regression coefficents, an approximation is necessary to obtain the variances of each elasticity. A first order Taylor expansion about the mean of the variables is used. In this case

$$f = \frac{c_{12}}{c_1 c_2} + 1 \qquad 1,2 \text{ from } (L,K,F)$$

so

$$\text{var}(f) = \text{var}\left(\frac{c_{12}}{c_1 c_2}\right).$$

The three variable Taylor expansion about the means of $c_{12}, c_1, c_2 (w_{12}, w_1, w_2$ respectively) is

$$f(c_1, c_2, c_{12})_{w_1, w_2, w_{12}} =$$

$$= f(w_{12}, w_1, w_2) + (c_1 - w_1)f_{c_1} + (c_2 - w_2)f_{c_2} + (c_{12} - w_{12})f_{c_{12}} \quad .$$

Since

$$var(f) = E(f - E(f))^2$$

where E is the expected value, the expected value of f is

$$E(f) = f(w_{12}, w_1, w_2).$$

From this

$$\begin{aligned} Var(f) &= E((c_1 - w_1)f_{c_1} + (c_2 - w_2)f_{c_2} + (c_{12} - w_{12})f_{c_{12}})^2 \\ &= E((c_1 - w_1)^2 f_{c_1}^2 + (c_2 - w_2)^2 f_{c_2}^2 + \\ &\quad (c_{12} - w_{12})^2 f_{c_{12}}^2 + 2(c_1 - w_1)(c_2 - w_2) f_{c_1} f_{c_2} + \\ &\quad 2(c_1 - w_1)(c_{12} - w_{12}) f_{c_1} f_{c_{12}} + \\ &\quad 2(c_2 - w_2)(c_{12} - w_{12}) f_{c_2} f_{c_{12}}) \\ &= var(c_1) f_{c_1}^2 + var(c_2) f_{c_2}^2 + var(c_{12}) f_{c_{12}}^2 + \\ &\quad 2cov(c_1, c_2) f_{c_1} f_{c_2} + 2cov(c_1, c_{12}) f_{c_1} f_{c_{12}} + \\ &\quad 2cov(c_2, c_{12}) f_{c_2} f_{c_{12}} \ . \end{aligned}$$

For this nonlinear function

$$\frac{\partial f}{\partial c_{12}} = \frac{1}{c_1 c_2} \qquad \frac{\partial f}{\partial c_1} = \frac{-c_{12}}{c_2 c_1^2} \qquad \frac{\partial f}{\partial c_2} = \frac{-c_{12}}{c_1 c_2^2} \ .$$

Others (Humphrey, 1975) have calculated the variance by assuming c_1 and c_2 are nonstochastic. They report

$$Var(f) = \frac{1}{(c_1 c_2)^2} var(c_{12}).$$

In Table 3, the elasticities of substitution with their variances for 1972 and 1976 are presented. The method of Humphrey tends to understate the variance in comparison with this method.

Since the elasticity estimators are MLE's with asymptotic normal distributions, it is easily tested if the elasticities for the two years are equal. The standard statistical test for

Table 3

Comparison of Variance
Elasticity of Substitution

Elas-ticity	Estimate 1972 (var:derived,Humph)	Estimate 1976 (var:derived,Humph)
σ_{Lk}	2.057(.549,.713)	-5.03(60.4,12.3)
σ_{Lf}	-5.203(73.3,12)	.327(.302,.63)
σ_{kf}	24.05(984,708)	52.5(3710,592)

equality of means requires equal variances. An F test was performed on the variances and in only two cases were they equal, so an asymptotic t-test was used.

Overall(1972) reports the following statistic

$$\frac{\bar{X}_1 - \bar{X}_2}{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^{\frac{1}{2}}}$$

where

$\bar{X}_i$ is the mean of the i^{th} distribution

S_i^2 is the variance of the i^{th} distribution,

which is distributed t with degrees of freedom

$$\frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{S_1^4}{n_1^2(n_1 - 1)} + \frac{S_2^4}{n_2^2(n_2 - 1)}} .$$

The elasticities of substitution with their standard deviations, the change in elasticities between 1972 and 1976, and the results of the t-test described above are shown in the first two rows of Table 4. Since the more positive an elasticity is, the easier the substitution is, positive changes in elasticity imply substitution has become easier, while negative changes imply substitution has become more difficult over the time period.

The first column of Table 4 reports the F-statistic of the Chow test, indicating a technology change has occurred. From the earlier discussion, firms under uncertainty should move to more elastic technologies with respect to fuel after the initial price shocks. This expected result is reported, where the elasticity between labor and fuel increased 5.53

Table 4

Elasticities of Substitution
1972, 1976

Chow Test[1]	No. Ind.	Cost Share	Elasticity[2] Labor, Capital	$\Delta\sigma_{LK}$[4]	t of Δ	Elasticity[2] Labor, Fuel	$\Delta\sigma_{LF}$[4]	t of Δ	Elasticity[2] Capital, Fuel	$\Delta\sigma_{KF}$[4]	t of Δ
1.95	298	all	2.057(.741)[3] -5.03(7.77)	-7.1	-16	-5.203(8.56) .327(.549)	5.5	11	24.05(31.4) 52.46(60.9)	28	7
.906	68	.3%-.7%	-.107(.303) -.800(.568)	-.69	-8.9	6.13(3.06) -1.70(3.02)	-7.8	-15	-23.2(14.2) 32.3(27.4)	55	15
.881	80	.7%-1%	2.99(.954) -.425(1.33)	-3.4	-18	-5.85(3.18) -1.91(.964)	3.9	11	18.7(13.1) 17.3(55.5)	-1.4	-.83
1.67	104	1%-3%	9.27(6.24) -1.58(1.32)	-11	-17	-23.7(38.2) 3.02(5.64)	27	7	56.3(76.9) 133(272)	77	2.8
.613	42	greater3%	26.5(29.3) 5.55(3.28)	-21	-4.6	10.8(3.42) 8.40(5.26)	-2.4	-2.5	162(206) 64.8(82.8)	-97	-2.8

1. Test measuring change in cost function coefficents.
2. First number of each pair is 1972, second value is 1976.
3. Number in parenthesis is standard deviation
4. Δ :1976 - 1972

and the elasticity between capital and fuel increased 28.41, and the t-scores of change indicate both of these changes are significant. This then allows the conclusion that firms have been able to use technology which gives them greater latitude in the substitution of labor and capital for fuel.

Whether firms with different intensities of fuel usage (different fuel cost shares) are equally able to use this technology change is studied next. The remainder of Table 4 presents subgroups of the industries grouped by cost share of fuel.

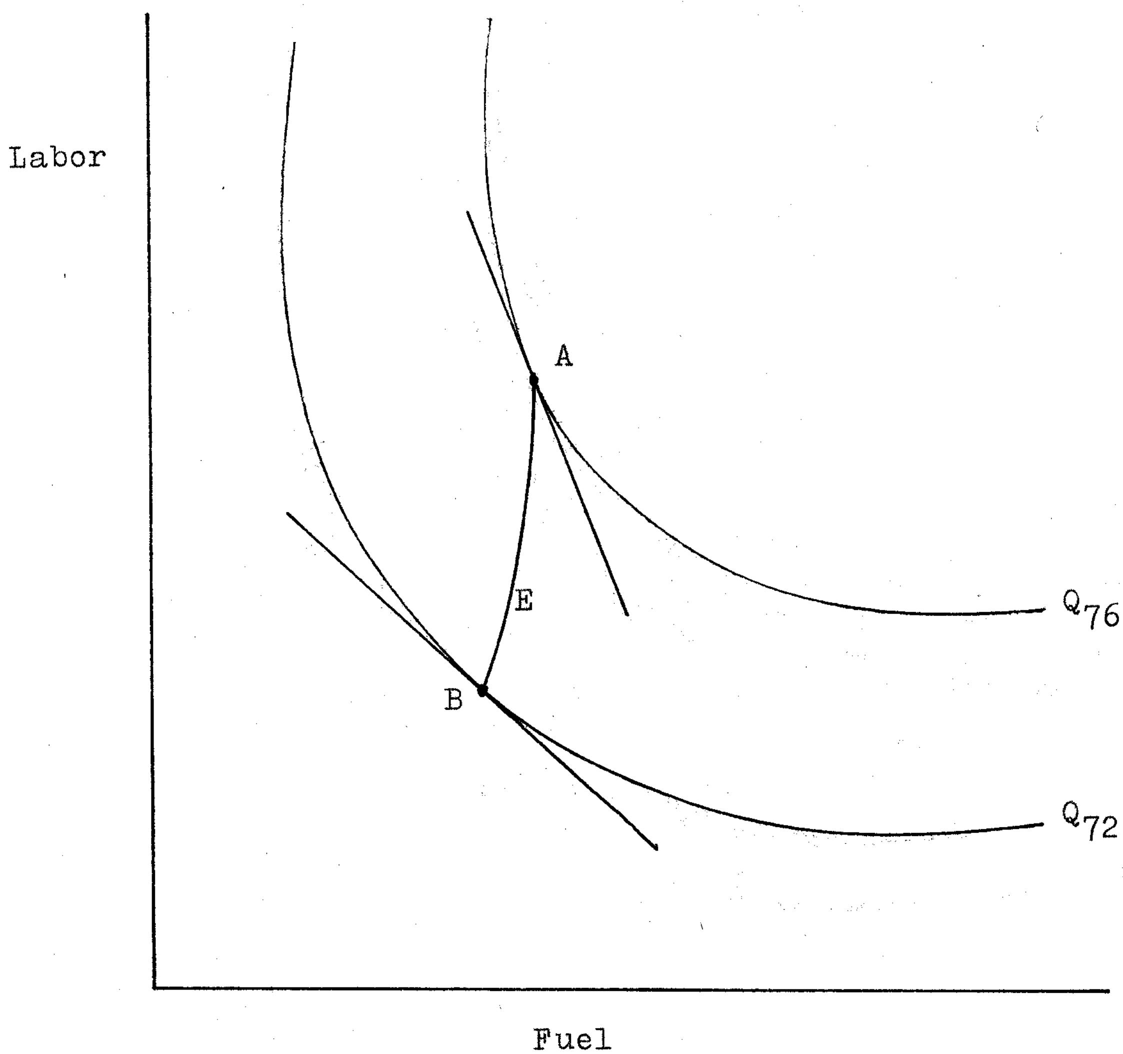
The first observation of this division is that the two digit industry code is a poor indicator of the fuel intensity for four digit industries under that code. As an example, of the 31 industries in group 20, Food and Kindred Products, 10 have cost shares between .3% and .7%, 9 between .7% and 1%, 9 between 1% and 3%, and 3 greater than 3%. Similar patterns are seen for other two digit codes.

Chow tests were performed on each group and only those industries with 1% to 3% fuel cost share changed technology between 1972 and 1976. In this group of 104 industries, the elasticity between fuel and labor increased 26.7, while the elasticity between capital and fuel increased 76.7. As with the study of all industries, it is easier for industries in this group to substitute between fuel and labor and capital under a new technology.

Industries with a stable technology are forced by the relative change in prices to a less fuel intensive position on a given isoproduct curve. Figure 1 diagrams a movement along an isoquant under changing relative prices and changing output. Between 1972 and 1976, an industry moves from isoquant Q_{72} to isoquant Q_{76} along the expansion path E. At point A the elasticity has changed from that of point B, where the firm was in 1972. It is not possible to tell

Figure 1

Isoquants 1972, 1976



from theory in which direction the elasticity has changed.

The groups of industries in this study reacted in a variety of manners. Firms with cost share between .3% and .7% experienced a decrease in the elasticity of substitution between labor and fuel and an increase in the elasticity between capital and fuel. Firms in the .7% to 1% cost share range experienced an increase in the elasticity of substitution between labor and fuel, while the capital, fuel elasticity remained the same. For industries with cost share greater than 3%, both elasticities with respect to fuel decreased. These results indicate that for firms with a stable technology, relative cost changes can effect any or all elasticities in either direction.

Between 1972 and 1976 the elasticity between labor and capital decreased without exception. Of equal interest the increased difficulty of substitution varies with the size of the fuel cost share of the firm. The larger the cost share, the more the elasticities have decreased. No similar pattern appears for the other two elasticity series.

IV. Conclusion

In this paper a three input translog cost function in capital, labor and fuel was estimated. For 298 U.S. manufacturing industries, price of labor, expenditure per worker, price of fuel, expenditure per unit of energy(with minor alteration due to data problems) and price of capital(value added minus expenditure on labor divided by total assets) were calculated for the years 1972 and 1976.

A significant change in the cost function occurred between 1972 and 1976, hence a change in technology occurred. From the earlier discussion, it was then expected that this change would be to a technology with greater elasticity for the fuel, capital and fuel, labor pairs. This in fact occurred. The sample was divided according to fuel cost shares and it was found that only one subgroup was using a different technology. For this group of 104 industries with cost shares between 1% and 3% the technology change was again in the expected direction with both the substitution between fuel and labor and between capital and fuel becoming easier.

In other subgroups, relative price changes forced movement along isoquants to less fuel intensive input combinations was the only movement. No general pattern of elasticity change was observed.

Industy wide and in each subgroup, the ability to substitute labor and capital decreased from 1972 to 1976. This decrease was largest for firms with largest fuel cost share and smallest for firms with the smallest cost share.

In this study, two types of industry groups appear. The first are those industries using a new technology and having an increased ease in substituting between fuel and labor and capital. The second set of industries have not moved to a new technology, but still experience elasticity changes due to forced movements along their isoproduct curve because of relative price changes. From these results, firms with fuel cost share in the range 1% to 3% are of the first type, while other industries are of the second type.

References

Allen, R.G.D. <u>Mathematical Analysis for Economics</u>. London: MacMillan, 1938.

Arrow, K., Chenery, H., Minhas, B., and Solow, R. "Capital-Labor Substitution and Economic Efficency." <u>Review of Economics and Statistics</u> 18(1961): 225-50.

Berndt, E.R. and Wood, D.O. "Technology, Prices and the Demand for Energy." <u>Review of Economics and Statistics</u> 57(August 1975): 259-69.

Chow, G.C. "Tests of Equality between Sets of Coefficents in Two Linear Regressions." <u>Econometrica</u> 28(July 1960): 591-605.

Christensen, L.R., Jorgenson, D.W., Lau, L.J. "Conjugate Duality and the Transcendental Logarithmic Production Function." <u>Econometrica</u> 39(July 1971): 255-56.

Cobb, L.W. and Douglas, P.H. "A Theory of Production." <u>American Economic Review: Papers and Proceedings</u> 18(1928): 139-65.

Guilkey, D.K. and Lovell C.A.K. "On the Flexibility of the Translog Approximation." <u>International Economic Review</u> (forthcoming).

Humphrey, D.B. and Moroney, J.R. "Substitution amoung Capital, Labor and Natural Resource Products in American Manufactures." <u>Journal of Political Economics</u> 83(1975): 57-83.

Overall, J. and Klett C. Applied Multivariate Analysis. New York:McGraw-Hill, 1972.

Schlesser, M. Energy in the Economy. London:The MacMillan Press LTD, 1978.

Shabecoff, P. "To Save Energy, More People are Being Hired." "The Cleveland Plain Dealer" February 23, 1980.

Shepard, R.W. Cost and Production Functions. Princeton, N.J.:Princeton University Press, 1953.

Thursby, J.G., and Lovell, C.A.K. "An Investigation of the Kmenta Aproximation to the CES Function." International Economic Review 19(June 1978): 363-77.

U.S. Bureau of the Census. Census of Manufactures, 1972. Vol. 1 and Vol. 5. Washington:Government Printing Office, 1974.

_____ Annual Survey of Manufactures:1971, 1976. Washington: Government Printing Office, 1973, 1978.

Uzawa, H. "Production Function with Constant Elasticities of Substitution." Review of Economic Studies 29(1962): 291-99.